

4. MATCH THE COLUMN:

Column - I

- (A) Ice formation in a lake
 (B) The mode not associated with solids
 (C) Water heated in a round bottomed flask
 (D) Land breeze

Column - II

- (P) Molecular transfer leads to uniform heat content
 (Q) Heat transfer by convection
 (R) Time taken is inversely proportional to ambient temperature.
 (S) Process of heat transfer by conduction

5. A planet is at an average distance d from the sun, and its average surface temperature is T . Assume that the planet receives energy only from the sun, and loses energy only through radiation from its surface. Neglect atmospheric effects.

If $T \propto d^{-n}$, the value of n is :

- (A) 2 (B) 1 (C) $1/2$ (D) $1/4$

6. A system S receives heat continuously from an electrical heater of power 10 W. The temperature of S becomes constant at 50°C when the surroundings temperature is 20°C . After the heater is switched off, S cools from 35.1°C to 34.9°C in 1 minute.

The heat capacity of S is :

- (A) $100\text{ J}^\circ\text{C}$ (B) $300\text{ J}^\circ\text{C}$ (C) $750\text{ J}^\circ\text{C}$ (D) $1500\text{ J}^\circ\text{C}$

ELASTICITY

Section - 5

The **deforming force** is the external force applied to a body which tends to change the size or shape of the body. Under the action of deforming force, a body opposes any change in its shape and size due to the net effect of internal molecular forces. The resulting force which opposes the deformation is known as restoring force.

Illustration the concept :**When one tries to stretch a spring**

- The force acting on the spring (F_1) is the deforming force.
- The force exerted by the spring to oppose the deforming action is the restoring force (F_2).
- By Newton's third Law, we can say that F_1 and F_2 are equal and opposite forces ; forming an Action- Reaction pair.

The property of a body due to which it opposes the action of the deforming forces is called as **Elasticity**.

- A material is said to be elastic if it returns back to its original shape or size, when the deforming forces are removed. Plastic materials on the other hand, remain permanently distorted when the deforming forces are removed.
- There are a number of elastic moduli (young's modulus, bulk modulus, shear modulus) that measure the response of an elastic solid which is acted upon by the deforming forces.
- The deforming force is measured (described) in terms of a physical quantity, the stress, that it develops in the body. The deformation of the solid is described in terms of a physical quantity the strain, that is created in the body as a result of deformation force.
- First, we will define the mathematical meaning of the stress and strain.

Stress and Strain

Longitudinal Stress and Longitudinal Strain :

Consider a rod (or a string) of length ' ℓ ' and cross-sectional area A . Suppose that a force F is applied at the ends parallel to the length so as to stretch the rod by $\Delta\ell$

The rod is said to be under tension. The tensile stress is defined as the ratio of the force of tension to the cross-sectional area.

$$\text{Tensile stress} = F/A$$

The resulting tensile strain is defined as the ratio of change in length to the original length :

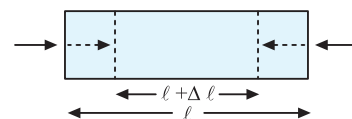
$$\text{Tensile strain} = \Delta\ell/\ell$$

If the ends of a rod are subjected to equal pushes, the rod is said to be under compression.

$$\text{Compressive stress} = F/A$$

$$\text{Compression strain} = \Delta\ell/\ell$$

Tensile and Compressive Stress and Strains are also known as Longitudinal stress and Longitudinal strains.



Illustrating the concept :

A bob of mass m is suspended from a light steel wire of length ℓ and radius r .

Assume the mass of the wire to be negligible.

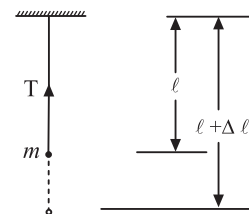
Let T be the tension developed in the steel wire. As the bob is stationary,

$$T = mg$$

Longitudinal stress = T/A

$$= mg/\pi r^2$$

If the length of wire increases by $\Delta\ell$, then the longitudinal strain = $\Delta\ell/\ell$



Volume stress and Volume Strain :

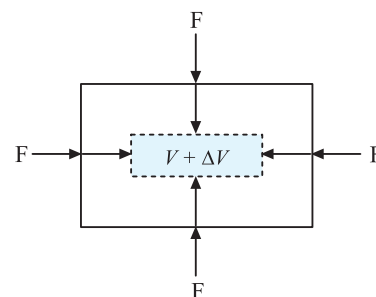
Sometimes, the deforming forces acting on the body tend to change its volume.

If the deforming force changes the volume of a body from V to $V + \Delta V$ (ΔV is negative if volume decreases), we define volume strain as the ratio of change in V to V :

$$\text{i.e. volume strain} = \frac{\Delta V}{V}$$

The stress associated with the volume strain is called as volume stress. It equals the deforming forces acting on a unit area. It is usually called as pressure

$$P = F/A$$



Illustrating the concept :

A cube is subjected to pressure of $5 \times 10^5 \text{ N/m}^2$. Each edge of the cube is shortened by 1%.

$$\text{Volume stress} = P = 5 \times 10^5 \text{ N/m}^2$$

volume strain = ?

Let ' ℓ ' be the length of cube (underformed)

$$\Rightarrow \quad \text{Distorted length} = \left(\frac{99}{100} \ell \right)^3 \quad \Rightarrow \quad \Delta V = V_f - V_i$$

$$\Delta V = \left(\frac{99}{100} \ell \right)^3 - \ell^3$$

$$\text{or} \quad \text{volume strain} = \frac{\Delta V}{V_i} = \left(\frac{99}{100} \right)^3 - 1$$

$$\approx -0.03$$

Shear Stress and Shear Strain :

If the deforming force tries to change the shape of a body, a shear stress is developed in the body. In the diagram, we see a force F parallel to the surface $ABCD$ of area A , deforming the rectangular area $ABPQ$ into a parallelogram.

[Let $AA' = \Delta x$ and $AQ = y$]

Hence due to F , the shape of the body is altered.

$$\text{Shear stress} = \frac{\text{shearing force}}{\text{area of } ABCD}$$

$$= \frac{F}{A}$$

Shear strain is defined as :

$$\text{Shear strain} = \frac{\Delta x}{y} = \tan \theta = \theta \quad (\theta \text{ is very small})$$

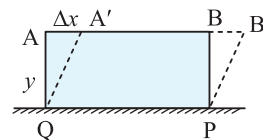
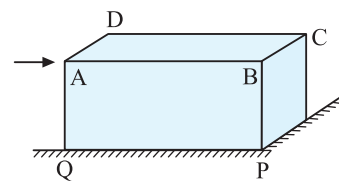
Illustrating the concept :

A block of weight 15 N slides across a horizontal table, the coefficient of sliding friction = 0.4. The area of the block in contact with the table is 0.05 m^2 .

$$\text{Shear stress} = \mu mg / A$$

$$= \frac{0.4 \times 15}{0.05}$$

$$= 120 \text{ N/m}^2$$



Hooke's Law :

If the deforming forces are within a limit (known as elastic limit), the stress created in the body is proportional to the resulting strain

$$i.e. \text{ stress} \propto \text{strain}.$$

The ratio $\frac{\text{stress}}{\text{strain}}$ is known as modulus of Elasticity

According to various types of stresses, we have three moduli of elasticity.

1. Young's Modulus of Elasticity (Y)

$$Y = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}}$$

$$Y = \frac{F / A}{\Delta \ell / \ell}$$

$$\Rightarrow Y = \frac{F \ell}{A \Delta \ell}$$

The following forms of the result must be carefully noted

$$\Delta \ell = \frac{F \ell}{A Y} \quad \text{and} \quad F = YA \frac{\Delta \ell}{\ell}$$

2. Bulk Modulus (K)

$$K = - \frac{\text{volume stress}}{\text{volume strain}}$$

$$\Rightarrow K = \frac{-p}{\Delta V / V}$$

$$\Rightarrow K = - \frac{pV}{\Delta V}$$

➤ the reciprocal of K is known as modulus of compressibility.

3. Shear modulus (η)

$$\eta = \frac{\text{shear stress}}{\text{shear strain}}$$

$$\Rightarrow \eta = \frac{F / A}{\theta}$$

$$\eta = \frac{F}{A \theta}$$

η is also called as modulus of rigidity

Relation between Y , k and η :

The three moduli of elasticity are related as $\frac{3}{Y} = \frac{1}{\eta} + \frac{1}{3K}$

Elastic Energy

When an elastic body is deformed, work is done by the applied force. This work is stored as elastic potential energy and is released when the body returns back to its original shape or size.

We have already come across an example of elastic potential energy in case of a compressed or stretched spring.

The general expression for the elastic potential energy in a unit volume of a deformed body is given by :

Elastic energy stored per unit volume

$$= 1/2 \text{ (stress) (strain)}$$

$$= 1/2 \text{ (modulus of elasticity) (strain)}^2$$

In the case of a longitudinal stress (compressive or tensile)

$$\frac{\text{Energy stored}}{\text{volume}} = \frac{1}{2} Y \left(\frac{\Delta \ell}{\ell} \right)^2$$

$$\Rightarrow \text{total energy stored} = \left[\frac{1}{2} Y \left(\frac{\Delta \ell}{\ell} \right)^2 \right] [\text{volume of the body}]$$

$$= \frac{1}{2} Y \left(\frac{\Delta \ell}{\ell} \right)^2 \cdot (A\ell) \quad (V = A \cdot \ell)$$

Total energy = $\frac{1}{2} \frac{YA}{\ell} \cdot (\Delta \ell)^2$. Compare it with the energy stored in a spring = $1/2 kx^2$.

Also, $Y = \frac{F\ell}{A\Delta \ell}$ Substituting in the above expression,

$$\text{Total energy} = \frac{1}{2} F \Delta \ell$$

Torsion

Consider a cylinder whose upper end is rigidly fixed and the other end is twisted through an angle θ about the axis of the cylinder. The twisted cylinder exerts a restoring torque given by :

$$\tau = \frac{1}{2} \frac{\pi \eta r^4}{\ell} \theta$$

where η is the modulus of rigidity, r is the radius and ℓ is the length of the cylinder.

If we express torque as $\tau = C\theta$

where $C = \frac{\pi \eta r^4}{2\ell}$ is known as the torsional rigidity.

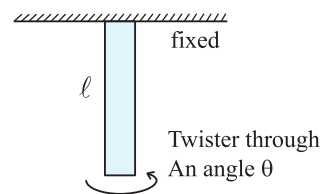


Illustration - 29 (a) A metal wire 75 cm long and 0.13 cm in diameter stretches 0.035 cm, when a load of 8 kg is hung on its ends. Find the stress, strain and Young's modulus.

- (b) A solid cylindrical steel column is 4m long and 9 cm in diameter. What will be its decrease in length when carrying a load of 80, 000 kg ? $Y = 1.9 \times 10^{11} \text{ N/m}^2$.
- (c) A box shaped piece of gelatine dessert has a top area of 15 cm^2 and a height of 3 cm. When a shearing force of 0.5 N is applied to the upper surface, the upper surface is displaced 4.0 mm relative to the bottom surface. What are the shearing stress, shearing strain and shear modulus ?
- (d) Compute the volume change of a solid copper cube, 40 mm on each edge, when subjected to a pressure of $2 \times 10^7 \text{ N/m}^2$. The bulk modulus of copper is $1.25 \times 10^{11} \text{ N/m}^2$.

SOLUTION :

$$\begin{aligned} \text{(a) Stress} &= \frac{F}{A} = \frac{8 \times 9.8}{\pi (6.5 \times 10^{-4})^2} \\ &= 5.9 \times 10^7 \text{ N/m}^2 \\ \text{Strain} &= \frac{\Delta \ell}{\ell} = \frac{0.035}{75} = 4.67 \times 10^{-4} \\ Y &= \frac{\text{stress}}{\text{strain}} = \frac{5.91 \times 10^7}{4.67 \times 10^{-4}} \\ &= 1.27 \times 10^{11} \text{ N/m}^2 \end{aligned}$$

$$\text{Shear strain} = \theta = \frac{\Delta x}{y} = \frac{(\text{displacement})}{\text{height}}$$

$$= \frac{0.4}{3} = 0.133$$

$$\begin{aligned} \eta (\text{shear modulus}) &= \frac{\text{shear stress}}{\text{shear strain}} \\ &= \frac{333}{0.133} \\ &= 2500 \text{ N/m}^2 \end{aligned}$$

$$\begin{aligned} \text{(b) Cross section area of cylinder} &= \pi r^2 \\ &= \pi (0.045)^2 = 6.36 \times 10^{-3} \text{ m}^2 \\ \Delta \ell &= \frac{F \ell}{AY} = \frac{(8 \times 10^4 \times 9.8) 4}{(6.36 \times 10^{-3}) (1.9 \times 10^{11})} \\ &= 2.6 \times 10^{-3} \text{ m} \\ &= 2.6 \text{ mm} \end{aligned}$$

$$\begin{aligned} \text{(c) Shear stress} &= \frac{\text{tangential force}}{\text{Area of face}} = \frac{0.5}{15 \times 10^{-4}} \\ &= 333 \text{ N/m}^2 \end{aligned}$$

$$\begin{aligned} \text{(d) Volume stress} &= p = 2 \times 10^7 \text{ N/m}^2 \\ \text{volume strain} &= \frac{\Delta V}{V} = \frac{\Delta V}{(0.04)^3} \\ K (\text{bulk modulus}) &= \frac{-p}{\Delta v / v} \\ \Rightarrow \Delta V &= \frac{-pV}{K} = \frac{-(2 \times 10^7) (0.04)^3}{1.25 \times 10^{11}} \\ &= -1.024 \times 10^{-8} \text{ m}^3 \\ &= -10.24 \text{ mm}^3 \end{aligned}$$

Illustration - 30 A wire elongates by 1.0 mm when a load W is hanged from it. If this wire goes over a pulley and two weights W each are hung at the two ends, the elongation of the wire will be :

- (A) 0.5 m (B) 1.0 mm (C) 2.0 mm (D) 4.0 mm

SOLUTION : (B)

The tension in the wire will be same in both the cases.

Hence elongation will be same. B is the correct answer

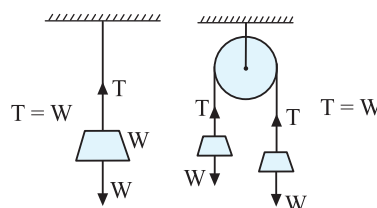


Illustration - 31 A heavy uniform rod is hanging vertically from a fixed support. It is stretched by its own weight. The diameter of the rod is :

- (A) smallest at the top and gradually increases down the rod
 (B) largest at the top and gradually decreases down the rod
 (C) uniform everywhere
 (D) maximum in the middle

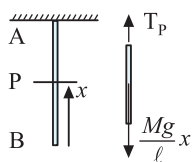
SOLUTION : (B)

Tension at a point P is :

$$T_P = \frac{Mg}{\ell} x$$

Tension is maximum at the top and zero at the free end

$$\Delta \ell = \frac{TL}{AY} \Rightarrow \Delta \ell \propto T$$



Hence the parts of the rod near the top end will be elongated more than the parts near the bottom. As volume of each part should remain same, (area of cross section) $\Delta \ell = \text{constant}$

$$\Rightarrow \text{Area} \propto \frac{1}{\Delta \ell} \Rightarrow \text{Area} \propto \frac{1}{T}$$

Hence the diameter will be least at the top and maximum at the bottom. (A is correct)

Illustration - 32 Assertion & Reasoning Type :

- (A) If Statement-1 is True, Statement-2 is True ; Statement-2 is a correct explanation for Statement-1
 (B) If Statement-2 is True, Statement-2 is True ; Statement-2 is NOT a correct explanation for Statement-1
 (C) If Statement-1 is True, Statement-2 is False
 (D) If Statement-1 is False, Statement-2 is True
 (a) **Statement 1** : Steel is more elastic than rubber.
Statement 2 : Under given deforming force, steel is deformed less than rubber.
 (b) **Statement 1** : A hollow shaft is found to be stronger than a solid shaft made of the same material.
Statement 2 : The torque required to produced a given twist in hollow cylinder is greater than that required to twist a solid cylinder of same size and material.

SOLUTION : (a) (A) (b) (A)

(a) $Y_{\text{steel}} > Y_{\text{rubber}}$

$$\frac{1}{Y_{\text{Steel}}} < \frac{1}{Y_{\text{rubber}}}$$

For a deforming force F,

$$\frac{F\ell}{AY_s} < \frac{F\ell}{AY_r}$$

$$\Delta \ell_s < \Delta \ell_r$$

(Reason is correct)

As it is difficult to deform steel, it can be said that steel offers greater opposition to changes in length. Hence steel is more elastic than rubber. The correct answer is (A).

(b) $\tau_1 = \frac{\pi \eta r^4}{2\ell}$ for solid shaft causing unit angular twist.

For hollow shaft of same material :

$$\tau_2 = \frac{\pi \eta (r_2^4 - r_1^4)}{2\ell} \text{ where } \pi (r_2^2 - r_1^2) \ell = \pi r^2 \ell$$

(for equal volume of material)

$$= \pi \eta \frac{(r_2^2 - r_1^2)(r_2^2 + r_1^2)}{2\ell} = \frac{\pi \eta r^4}{2\ell} (r_2^2 + r_1^2)$$

$$= \tau_1 \left(1 + \frac{2r_1^2}{r^2} \right)$$

$$\Rightarrow \tau_2 > \tau_1$$

$\Rightarrow$ The correct answer is (A).

Illustration - 33 The length of a metal wire is ℓ_1 when the tension in it is T_1 and is ℓ_2 when the tension is T_2 . The natural length of the wire is :

- (A) $\frac{\ell_1 + \ell_2}{2}$ (B) $\sqrt{\ell_1 \ell_2}$ (C) $\frac{\ell_1 T_2 - \ell_2 T_1}{T_2 - T_1}$ (D) $\frac{\ell_1 T_2 + \ell_2 T_1}{T_2 + T_1}$

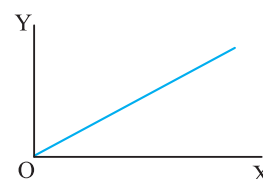
SOLUTION : (C)

Let the free length = ℓ

$$\Delta \ell = \frac{T \ell}{AY} \Rightarrow \frac{\ell_1 - \ell}{\ell_2 - \ell} = \frac{T_1}{T_2} \Rightarrow \ell = \frac{\ell_1 T_2 - \ell_2 T_1}{T_2 - T_1}$$

Illustration - 34 A student plots a graph from his readings on the determination of Young's modulus of a metal wire but forgets to put the labels. The quantities on X and Y-axes may be respectively :

- (A) weight hung and length increased
(B) stress applied and length increased
(C) stress applied and strain developed
(D) length increased and the weight hung



SOLUTION : (ABCD)

- (A) For a straight line graph, the two quantities should be proportional

$$\Delta \ell = \frac{F \ell}{AY} \Rightarrow F \propto \Delta \ell$$

Where F = weight hung.

Hence A is correct

- (B) Stress = $F/A = Y$ (Strain)

$$= \frac{Y}{\ell} \Delta \ell$$

$\Rightarrow$ Stress $\propto \Delta \ell$. Hence B is correct

$$(C) Y = \frac{\text{Stress}}{\text{Strain}}$$

$$\Rightarrow \text{stress} \propto \text{strain}$$

Hence C is correct

- (D) Same as A. This is also correct.

Illustration - 35 A load of 981 N is suspended from a steel wire of radius 1 mm. What is the maximum angle through which the wire with the load can be deflected so that it does not break when the load passes through the equilibrium position ? Breaking stress is $7.85 \times 10^8 \text{ N/m}^2$.

SOLUTION :

Let θ = maximum angular deflection.

$$\Rightarrow \frac{1}{2} mv^2 = mg(\ell - \ell \cos \theta)$$

$$\text{and } T - mg = mv^2/\ell$$

$$\Rightarrow T = 3mg - 2mg \cos \theta$$

For maximum θ , T/A must be $7.85 \times 10^8 \text{ N/m}^2$.

$$\frac{3mg - 2mg \cos \theta}{\pi (10^{-3})^2} = 7.85 \times 10^8$$

$$\Rightarrow \cos \theta = 0.243 \Rightarrow \theta = \cos^{-1}(0.243)$$

Illustration - 36 A copper rod of length 2 m is stretched by 5mm. If the energy stored in the stretched rod is converted into heat, calculate the rise in the temperature of the rod. $Y_{\text{copper}} = 1.2 \times 10^{11} \text{ N/m}^2$, $S_{\text{Cu}} = 0.1 \text{ cal/gm/}^\circ\text{C}$, $d_{\text{Cu}} = 9.09 \text{ g/cc}$.

SOLUTION :

Total elastic energy stored

$$= 1/2 (\text{stress}) (\text{strain}) \text{ volume}$$

$$= 1/2 Y (\text{strain})^2 \text{ volume}$$

$$= 1/2 Y (\Delta \ell / \ell)^2 V$$

Energy stored = heat created

$$1/2 Y (\Delta \ell / \ell)^2 V = (ms\theta) 4.2 \text{ J/cal}$$

$$[1 \text{ cal} = 4.2 \text{ J}]$$

$$1/2 Y (\Delta \ell / \ell)^2 V = (V ds \theta) 4.2 \text{ J/cal}$$

$$\Rightarrow \theta = \frac{1}{2} Y \left(\frac{\Delta \ell}{\ell} \right)^2 \frac{(4.2)^{-1}}{ds}$$

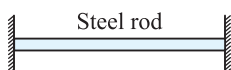
$$= \frac{(1.2 \times 10^{11})}{2} \left(\frac{5 \times 10^{-3}}{2} \right)^2 \frac{(4.2)^{-1}}{(9 \times 10^3) (0.1 \times 10^3)}$$

$$[S = 0.1 \times 10^3 \text{ cal/kg}^\circ\text{C}]$$

$$\Rightarrow \theta = 0.099^\circ\text{C}$$

Illustration - 37 A steel rod of cross-sectional area 0.5 mm^2 is held between two fixed supports. If the tension in the rod is negligible and it is just taut at 20°C , determine the tension when the temperature falls to 0°C . $Y_{\text{steel}} = 21 \times 10^{10} \text{ N/m}^2$, $\alpha_{\text{steel}} = 12 \times 10^{-6} / ^\circ\text{C}$. Assume the distance between the supports remains same.

SOLUTION :



As the temperature drops ($20^\circ \rightarrow 0^\circ\text{C}$) the rod tends to contract.

As the ends of the rod are rigidly fixed, its length can not change. The rigid supports now have a tendency to pull the rod creating a tensile stress.

Let $\Delta \ell_1$ = change in length due to temperature drop

$\Delta \ell_2$ = change in length due to tensile stress.

($\Delta \ell_1$ is -ve and $\Delta \ell_2$ is +ve)

As there is no net change in length,

[The stress created like this is called as Thermal stress]

$$\Rightarrow \Delta \ell_1 + \Delta \ell_2 = 0$$

$$(-\ell \alpha \theta) + T \ell' A Y = 0 \quad [T = \text{tension in rod}]$$

$$\Rightarrow T = Y A \alpha \theta$$

$$= (21 \times 10^{10}) (0.5 \times 10^{-6}) (12 \times 10^{-6}) (20)$$

$$T = 25.2 \text{ N}$$

Illustration - 38 A cylinder of length 1.5 m and diameter 4 cm is fixed at one end. A tangential force of $4 \times 10^5 \text{ N}$ is applied at the other end. Calculate the twist produced in the cylinder if the modulus of rigidity is $6 \times 10^{10} \text{ N/m}^2$.

SOLUTION :

In equilibrium, applied torque = restoring torque.

$$Fr = \frac{\pi \eta r^4}{2 \ell} \theta$$

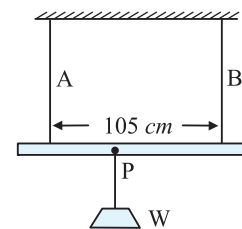
$$\Rightarrow \theta = \frac{2 \ell Fr}{\pi \eta r^4} = \frac{2 \times 1.5 \times 4 \times 10^5}{\pi (6 \times 10^{10}) (0.02)^3}$$

$$= 0.796 \text{ radians } (\theta \approx 45.5^\circ)$$

Illustration - 39 A rod 1.05 m long, whose weight is negligible, is supported at its ends by wires A and B of equal lengths as shown. The cross-section of A is 1 mm^2 and that of B is 2 mm^2 . The Young's modulus for wire A is $2.4 \times 10^{11} \text{ N/m}^2$ and for B is $1.6 \times 10^{11} \text{ N/m}^2$.

At what point along the rod should a weight be suspended in order to produce

(a) equal stress in A and B? (b) equal strain in A and B?



SOLUTION :

(a) Let x = distance of the point P where an weight is suspended from wire A.

Let F_1 and F_2 be the tensions in A and B.

Balancing torques about P, $F_1 x = F_2 (105 - x)$

As the rods have equal stress

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \Rightarrow 2F_1 = F_2$$

Solving for x we get : $\frac{105 - x}{x} = \frac{1}{2}$

$$\Rightarrow x = 70 \text{ cm} \quad (\text{from A})$$

(b) One equation is same i.e. $F_1 x = F_2 (105 - x)$

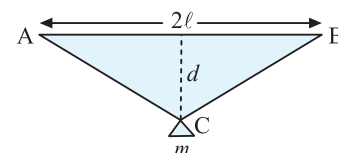
$$\text{equal strain} \Rightarrow \frac{Y_1 A_1}{F_1} = \frac{Y_2 A_2}{F_2}$$

$$\Rightarrow \frac{F_1}{F_2} = \frac{2.4}{1.6} \times \frac{1}{2} = \frac{3}{4}$$

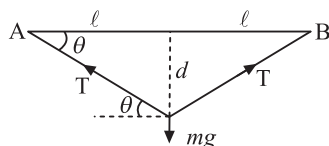
$$\Rightarrow \frac{105 - x}{x} = \frac{3}{4}$$

$$\Rightarrow x = 60 \text{ cm} \quad (\text{From A}).$$

Illustration - 40 A wire of radius r stretched without tension, along a straight line is tightly fixed at A and B. A mass m is suspended from the mid point of the wire. Due to the weight of mass, the wire is pulled into the shape ACB. Find the depression 'd' in the wire. The length of the wire is 2ℓ and its young's modulus is Y .



SOLUTION :



Let T be the tension in the wire.

$$2T \sin \theta = mg$$

$$\Rightarrow T = \frac{mg}{2 \sin \theta}$$

$$\text{Stress} = \frac{T}{A} = \frac{mg}{2A \sin \theta} = \frac{mg}{2\pi r^2 \sin \theta}$$

$$\text{Strain} = \frac{\Delta \ell}{\ell} = \frac{\sqrt{d^2 + \ell^2} - \ell}{\ell}$$

$$= \left(1 + \frac{d^2}{\ell^2} \right)^{1/2} - 1 \approx \frac{d^2}{2\ell^2}$$

$$\text{As } \sin \theta = \frac{d}{\sqrt{d^2 + \ell^2}} \approx \frac{d}{\ell} \quad (\text{neglecting } d^2, \text{ because } d^2 \ll \ell^2)$$

$$\text{Stress} = \frac{mg \ell}{2\pi r^2 d}$$

$$\Rightarrow Y = \frac{\text{stress}}{\text{strain}} = \frac{\frac{mg \ell}{2\pi r^2 d}}{\frac{d^2}{2\ell^2}}$$

$$\Rightarrow Y = \frac{mg \ell^3}{\pi r^2 d^3} \Rightarrow d = \ell \left(\frac{mg}{\pi r^2 Y} \right)^{1/3}$$

IN-CHAPTER EXERCISE - C

- A steel wire 40 m long and 2 mm in diameter hangs vertically down from the roof of a house.
 - What maximum load can be suspended from this wire without its breaking ?
 - How much will this wire stretch under a weight of 70 kg ?
 - Will there be a permanent set when the weight is removed ?

The breaking stress for steel is $7.8 \times 10^8 \text{ N/m}^2$, $Y_{\text{steel}} = 2.16 \times 10^{11} \text{ N/m}^2$ and elastic limit for steel is $2.94 \times 10^8 \text{ N/m}^2$.
- When the weight on a string is changed from 3.0 kg to 5.0 kg, the elongation changes from 0.61 mm to 1.02 mm. How much work is done during this extension of the string ? Find the Y of material of string, if it is 1 m long and has a cross-sectional area of $0.4 \times 10^{-4} \text{ m}^2$.
- A uniform rigid bar of mass 100 kg is supported horizontally by three equal vertical wires A, B and C each of initial length 1 m and cross-sectional area 1 mm^2 . B is a copper wire passing through the centre of bar and A and C are steel wires and are arranged symmetrically on each side of B. If $Y(\text{Cu}) = 1.5 \times 10^{11} \text{ N/m}^2$ and Y of steel $= 2 \times 10^{11} \text{ N/m}^2$, find the tension in each wire and the extension.
- A sphere of radius 10 cm and mass 25 kg is attached to the lower end of a steel wire which is suspended from the ceiling of the room. The point of support is 521 cm above the floor. When the sphere is set swinging as a simple pendulum, its lowest point just grazes the floor. Calculate the velocity of the ball at its lowest position. Y of steel is $2 \times 10^{11} \text{ N/m}^2$, the length of wire is 5 m and radius of steel wire is 0.05 cm.
- A steel shaft of radius 5 cm and length 4 m is transmitting power of $1 \times 10^3 \text{ Watts}$ at 240 rpm. If the relative twist between the ends of the shaft rod is $1.07 \times 10^{-3} \text{ radian}$, calculate the modulus of rigidity of steel.

Choose the correct alternative. Only One choice is Correct.

- A thick copper rope of density $1.5 \times 10^3 \text{ kg/m}^3$ and Young's modulus $5 \times 10^6 \text{ N/m}^2$, 8 m in length is hung from the ceiling of a room. The increase in its length due to its own weight is :
 - $9.6 \times 10^{-5} \text{ m}$
 - $19.2 \times 10^{-7} \text{ m}$
 - $9.6 \times 10^{-2} \text{ m}$
 - 9.6 m
- A wire of cross-sectional area a , length ℓ and young's modulus Y is extended by an external force F . The total energy stored in the wire is :
 - $\frac{1}{2} Y \frac{F^2}{a^2}$
 - $\frac{1}{2} Y \frac{F^2}{a} \ell$
 - $\frac{1}{2} Y \frac{F^2}{a^2}$
 - $\frac{1}{2} \frac{F^2 \ell}{aY}$
- Consider an iron bar 100cm long and 10 cm^2 cross-section. By raising the temperature from 0°C to 100°C and holding it so that it is not permitted to expand or bend, the force developed is: ($1\text{N} = 10^5 \text{ dynes}$, $Y = 2 \times 10^{11} \text{ N/m}^2$, $\alpha = 5 \times 10^{-6}/^\circ\text{C}$)
 - 10^8 dynes
 - 10^9 dynes
 - 10^{10} dynes
 - 10^{14} dynes
- A metal rod of Young's modulus $2 \times 10^{10} \text{ N/m}^2$ undergoes an elastic strain of 0.06%. The energy per unit volume stored in Joule/metre⁻³ is :
 - 3600
 - 7200
 - 1800
 - 900
- A uniform rod AB is rotating in a horizontal circle about a vertical axis through A. Ratio of elastic potential energy density of the rod at A and at the center of rod is :
 - 16 : 9
 - 1 : 1
 - 4 : 1
 - 2 : 1

